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ANALYSIS OF STUDENTS SPECIALIZING AND GENERALIZING THINKING IN THREE-DIMENSIONAL GEOMETRY

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ABSTRACT

This research analyzes the thinking processes of students specializing and generalizing in solving problems related to three-dimensional geometry. The objective of this research is to clearly identify how the cognitive processes of specializing and generalizing assist students in comprehending and solving problems related to three-dimensional geometry. This study employed a descriptive qualitative method to investigate students' problem-solving processes in three-dimensional geometry. Two ninth-grade students were selected through purposive sampling based on their mathematical abilities. Data were collected through task-based interviews and analyzed using qualitative coding to identify key problem-solving strategies and cognitive processes based on their ability to demonstrate specializing and generalizing thinking. Data were collected through geometry problem-solving tasks and unstructured interviews, and analyzed using Stacey's cognitive framework of mathematical thinking. The result of study reveal that the students demonstrated two distinct forms of schematic representation in their problem-solving processes. Subject 1 employed a pure schematic representation and subject 2 utilized a mixed schematic representation. These two forms of representation enable students to identify patterns and apply the concepts they have learned in a broader context. The findings indicate that students exhibit distinct cognitive patterns in specializing and generalizing, with a predominant reliance on visual reasoning in spatial geometry problem-solving. While some students transition effectively between these two processes, others face difficulties in generalizing abstract concepts.

Keywords: Pure schematic representation, Mixed schematic representation, Specializing and Generalizing, Three-dimensional geometry

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INTRODUCTION

Mathematics is a discipline that not only serves as a means of calculation, but also as a means to train logical, systematic, and critical thinking. In the context of education, learning mathematics is not solely aimed at mastering formulas and algorithms, but also at developing a structured and analytical mindset in solving problems (Li, 2018). An effective mathematics learning process should provide space for students to explore, connect concepts, and reflect on the strategies used in problem solving (Hasan, 2024). Therefore, learning approaches that emphasize conceptual understanding and active student involvement are very important. One aspect that becomes the focus in this process is the development of mathematical thinking skills as the main foundation in forming a complete mathematical competence(Fajri, 2017).

The development of students' mathematical thinking is a central focus in mathematics education, beyond merely mastering computational techniques or problem-solving procedures (Sani, 2023). Stacey describes mathematical thinking as a structured and sequential process involving four key stages: specializing, generalizing, conjecturing, and convincing(Lorenza et al., 2024). Among these, the processes of specializing and generalizing are considered fundamental, as they shape how students build patterns, abstract knowledge, and transfer it across various contexts. Specializing refers to the process of identifying specific examples to understand mathematical properties, while generalizing involves extending those patterns to more abstract or generalized forms (Mason, 2013).

This dual mode of thinking is particularly essential in geometry, especially in spatial topics such as three-dimensional shapes. Mastery of spatial geometry covering concepts like cubes, prisms, pyramids, and spheres requires not only abstract thinking but also strong visual reasoning skills and the ability to mentally manipulate spatial relationships (Mukhlis et al., 2024). However, several studies have highlighted that many students struggle with fundamental spatial understanding. For instance, (Susanto, 2018) found that students frequently face difficulties visualizing three-dimensional objects.(Suwito, 2020) also reported that learners often fail to connect two-dimensional representations with their corresponding three-dimensional forms, resulting in misconceptions.

Field observations conducted at SMPN 20 Malang support these findings. Students were found to struggle when solving spatial geometry problems, as reflected in their written work and reinforced through direct interviews. Most students had difficulty identifying problem patterns, understanding given data, and deciding appropriate strategies. This implies a lack of structured cognitive processes especially in forming and applying mathematical generalizations. Therefore, introducing thinking strategies such as specializing and generalizing may provide a meaningful solution.

Specializing helps students begin with familiar examples, identify regularities, and clarify underlying properties (Pullin, 2005). Generalizing then allows them to apply these insights to more complex and abstract shapes, such as pyramids or spheres (Park & Kim, 2017).These strategies are not only powerful in problem-solving but also instrumental in developing higher-order thinking and abstraction (Lorenza, 2024).(Lorenza, 2024)(Lorenza, 2024)

While numerous studies have investigated mathematical thinking processes, limited research has specifically examined how specializing and generalizing are applied by students in the context of three-dimensional geometry. Most prior studies such as those by Lorenza et al. (2024) and Susanti et al. (2019) focused on arithmetic sequences or algebraic reasoning. Although these studies identified schematic thinking patterns (explicit and implicit), they mainly emphasized procedural strategies, with little attention to students' conceptual understanding or individual cognitive pathways(Susanti et al., 2019).

This study seeks to address that gap by exploring the actual process of specializing and generalizing in solving spatial geometry problems. It utilizes Stacey’s thinking framework and introduces a comparative analysis of students' schematic approaches categorized as pure schematic and mixed schematic representation. By focusing on students’ cognitive processes during problem-solving, this research highlights how different schematic forms reflect variations in students’ mathematical thinking.

The findings of this study are expected to provide meaningful contributions to mathematics education. For teachers, they offer insight into how students think and how visual and abstract reasoning can be supported in the classroom. For curriculum developers, the results may serve as a basis for integrating schematic-based reasoning tasks into geometry learning.

Thus, the objective of this study is to investigate how students utilize specializing and generalizing thinking processes in solving three-dimensional geometry problems, and how these thinking modes are reflected through different schematic representations. Understanding this can help improve learning strategies, enhance students' mathematical reasoning, and support the development of geometry curricula that align with students' cognitive needs.

METHOD

This study adopts a descriptive qualitative approach, specifically a case study design, to explore students' mathematical thinking processes, particularly specializing and generalizing, in solving spatial geometry problems. A qualitative case study was chosen as it enables an in-depth investigation of individual students' reasoning and the representations they construct while solving complex problems. This approach is appropriate because it focuses on understanding meaning, context, and the processes behind students' problem-solving behavior (Craswell, 2012).(Craswell, 2012)(Craswell, 2012)

The research was conducted at SMPN 20 Malang, involving ninth-grade students who had previously studied the topic of spatial geometry. Three students were initially selected based on purposive sampling, with selection criteria including: (1) having completed instruction in spatial geometry, (2) being able to solve geometry problems with observable signs of specializing and generalizing thinking, and (3) being able to communicate their reasoning clearly during interviews. These students were identified after a preliminary analysis of their written responses and classroom performance. From these three, two students were found to exhibit relevant characteristics aligned with the goals of the study, and one student was ultimately selected as the main subject for in-depth analysis.

The data collection procedures in this study consisted of two main stages. First, a problem-solving test was administered, in which the subject was asked to solve two spatial geometry problems specifically designed to elicit the processes of specializing and generalizing. These tasks required the student to explore spatial concepts through particular examples and then extend those findings into broader generalizations. Second, a semi-structured interview was conducted following the test to gain deeper insight into the subject’s thought process. The interview was unstructured in nature, allowing flexibility to follow up on responses and clarify how the student approached the problems using visual and schematic reasoning. All interviews were recorded and transcribed for further analysis. The instruments used in this study included a set of two spatial geometry problems and an interview guideline developed to identify indicators of specializing and generalizing thinking, based on Stacey’s theoretical framework. The data analysis in this study uses descriptive qualitative methods. Qualitative data analysis is a step to manage, organize, and filter data into units that can be grouped, then synthesize the data, and identify emerging patterns. The data analysis technique in this study uses the theory from Miles & Huberman. The stages of the analysis technique can be shown as follows (Sugiyono, 2013).

Table 1. Stages of qualitative data analysis

Analysis Stage	**Description**
Data Reduction	Sorting data based on indicators from specializing-generalizing thinking, where the results of the thinking are represented in the form of schemes, either directly or indirectly.

Data Presentation	The data presented is a description of the thinking patterns of students in specializing-generalizing when solving problems related to arithmetic sequences and series.
Conclusion drawing	The conclusion found a schematic representation in the results of the subjects' work and thoughts.

Based on the coding process involved manual categorization of student responses according to thematic indicators of specializing and generalizing (e.g., identifying patterns, making conjectures, forming general rules). Data were organized in matrices to compare the emergence of each indicator across tasks and interview segments. Although no specialized software was used, coding sheets and analytical memos were maintained throughout to ensure consistency and traceability. This methodological approach allows for a nuanced exploration of how middle school students engage with spatial geometry problems and express their reasoning through schematic representations. It also provides a basis for understanding the development of mathematical thinking, particularly specializing and generalizing, in real classroom contexts.

RESULT

The results of this study show that in solving the problem of spatial geometry, the two subjects involved showed a tendency to think mathematically through two types of schematic representations, namely pure schematic representation and mixed schematic representation. This representation reflects the process of thinking specializing and generalizing in understanding and solving geometry problems.

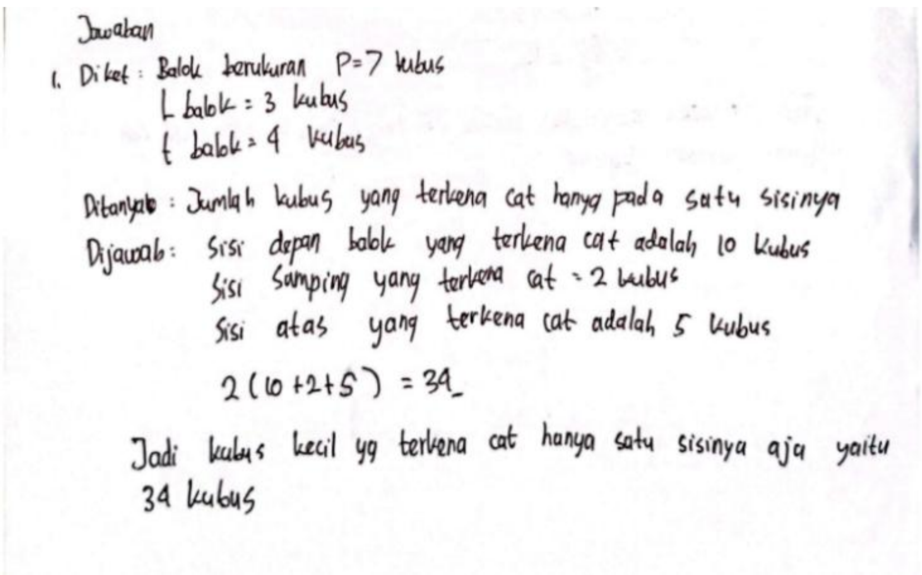
Jawaban
1. Diket : Balok berukuran P=7 kubus
L balok = 3 kubus
t balok = 4 kubus
Ditanya : Jumlah kubus yang terkena cat hanya pada satu sisinya
Dijawab : Sisi depan balok yang terkena cat adalah 10 kubus
Sisi samping yang terkena cat = 2 kubus
Sisi atas yang terkena cat adalah 5 kubus
2 (10 + 2 + 5) = 34
Jadi kubus kecil yg terkena cat hanya satu sisinya aja yaitu 34 kubus

Figure 3. Subject 1's Answer in Solving Geometry Problem Number 1

The first subject (SU1) showed a tendency to think that was classified as pure schematic representation. In the first problem, SU1 solved a problem related to building blocks of space composed of small cubes. He starts by identifying the dimensions of the block based on the number of cubes that make up it, namely seven cubes on the long side, three on the wide side, and four on the high side. From this visual analysis, SU1 determined the number of cubes that were exposed to paint on one side only. He noted that the front side has ten cubes, the side side two cubes, and the top side five cubes. Since the three sides have opposite side pairs, the number is multiplied by two, so that the final result is 34 cubes. This completion process shows that SU1 understands the structure of the building space and is able to implement a systematic and logical calculation strategy.

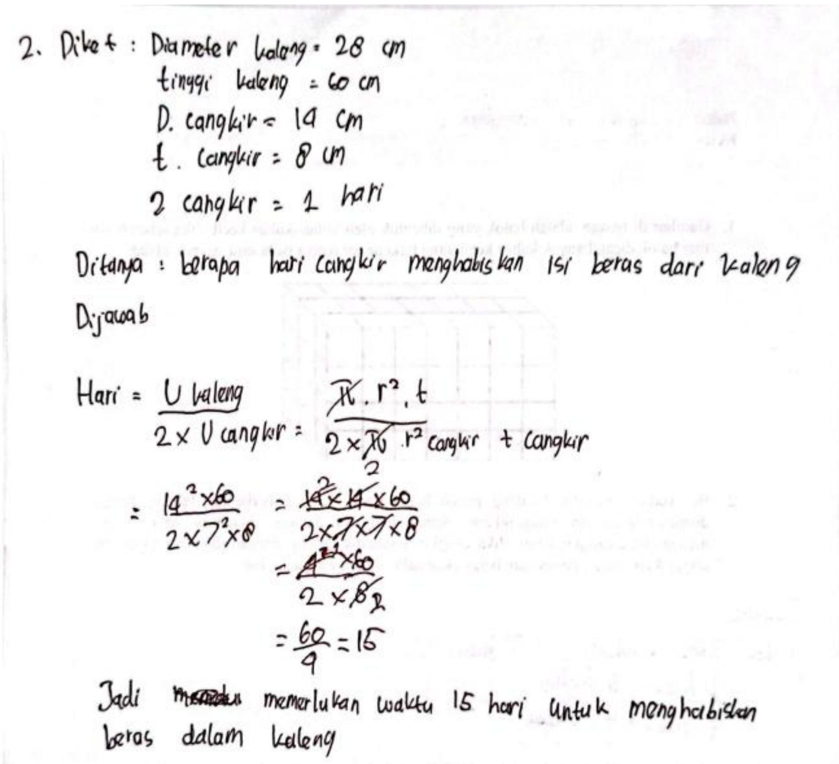

Figure 4. Subject 1's Answer in Solving Geometry Problem Number 2

In the second question, SU1 solves the problem by understanding the information in the question and developing an appropriate strategy. He used the tube volume formula to calculate the volume of the can, then divided it by twice the volume of the cup to find out how many days the contents of the can could be spent. The steps taken by SU1 reflect a structured mathematical flow of thinking: starting from understanding the problem, strategizing, substituting numbers into formulas, calculations, and drawing final conclusions. The interviews conducted showed that SU1 consciously formed a scheme in his mind to help the problem-solving process. The representations used are concise, direct, and reflect a deep understanding of the important elements of the problem at hand.

The following is a chart that shows the thinking ability of specializing-generalizing, which includes the process of pure schematic representation in solving geometry problems.

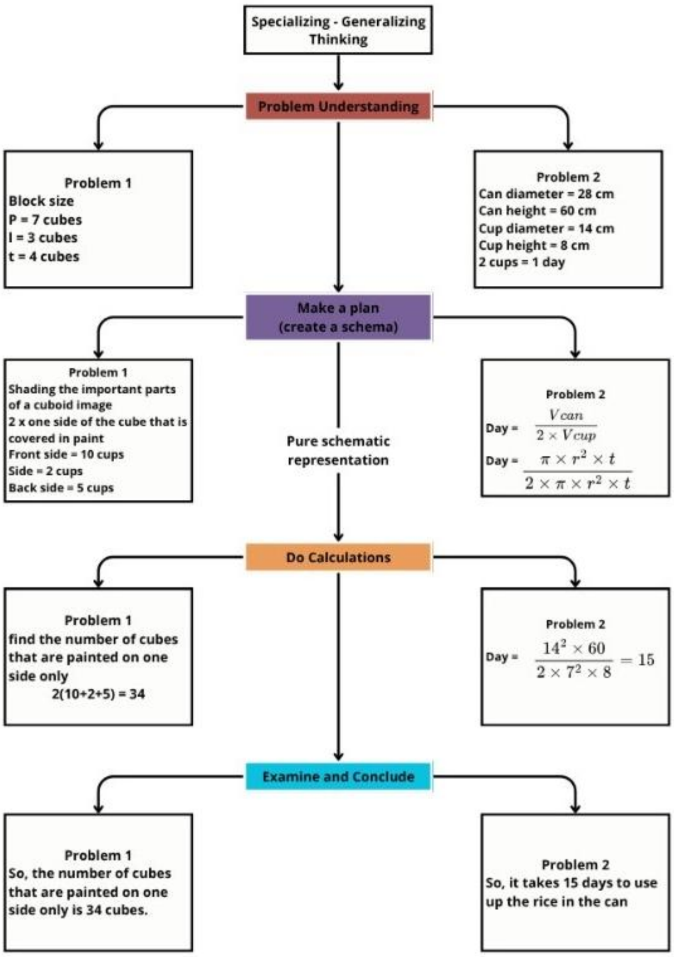

Figure 5. Diagram of the Specializing and Generalizing Thinking Process of SU1.

In contrast to SU1, the second subject (SU2) solved the problem with a mixed schematic representation approach. In the first problem, SU2 also identified the dimensions of the cube block and calculated the number of cubes exposed to paint on three sides. He used a similar method to SU1 and obtained the same result, which was 34 cubes. However, the SU2 approach is accompanied by visual sketches and shading to mark an important part of building a space. This suggests that SU2 relies on strong visual representations to build understanding, in addition to text or mathematical symbols. The results of the SU2 work in the following image.

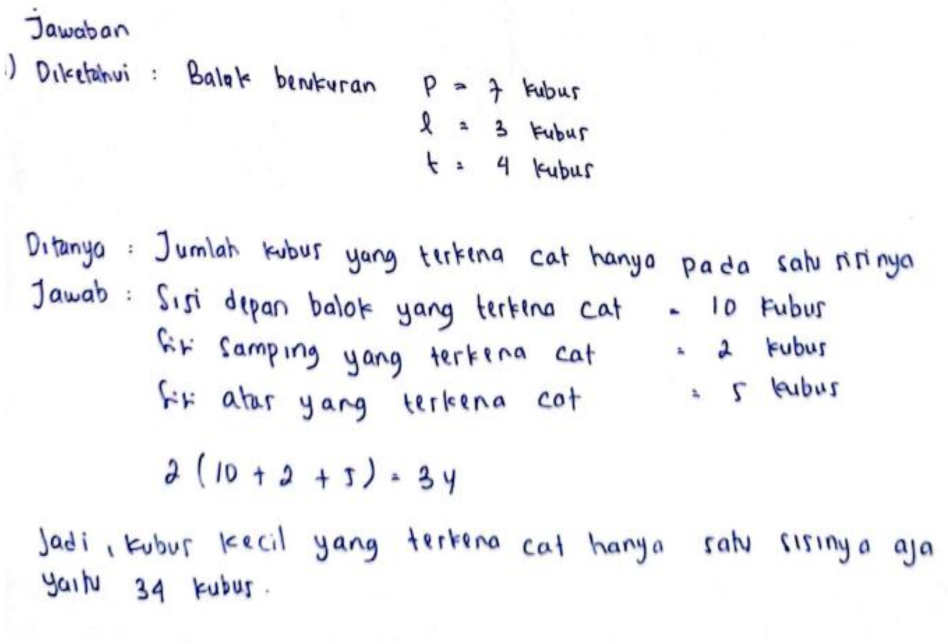

Figure 6. Subject 2's Answer in Solving Geometry Problem Number 1

In the second problem, SU2 drew a schematic of two objects, namely a can and a cup, then determined important parameters such as the diameter and height of each object. He calculated the volume of the two objects using the tube volume formula, then multiplied the volume of the cup by two to account for daily consumption. Next, the volume of the can is divided by the total volume of two cups to obtain the number of days it takes to finish the contents of the can. The results obtained show that SU2 is able to understand and model the situation into a mathematical form accordingly. SU2's thought process is reinforced through interviews, where he explains the steps of solving in a concise and logical manner, showing that his understanding develops from a combination of visualization and mathematical symbolism.

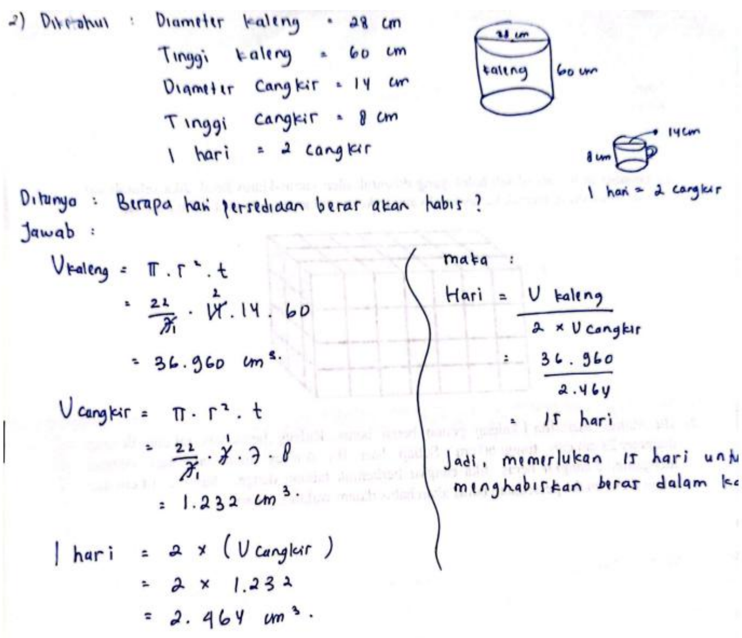

Figure 7. Subject 2's answer in solving geometry problem number 2.

The results from the interviews conducted on questions 1 and 2 show that the first step taken by SU2 was to read and understand the questions so that SU2 could gather information on what was known and what was being asked. The second step was for SU2 to find the appropriate schematic diagram to find alternative solutions to both problems. The third step involved performing calculations based on the chosen solution to solve both problems. The fourth step was for the student to correct the calculations made and write a conclusion based on the results of the calculations. The worksheets and explanations during the interview stage indicate that there was a process of mixed schematic representation carried out by SU1.

The following is a diagram that shows the ability to think specializing-generalizing, which includes the process of mixed schematic representation in solving geometry problems.

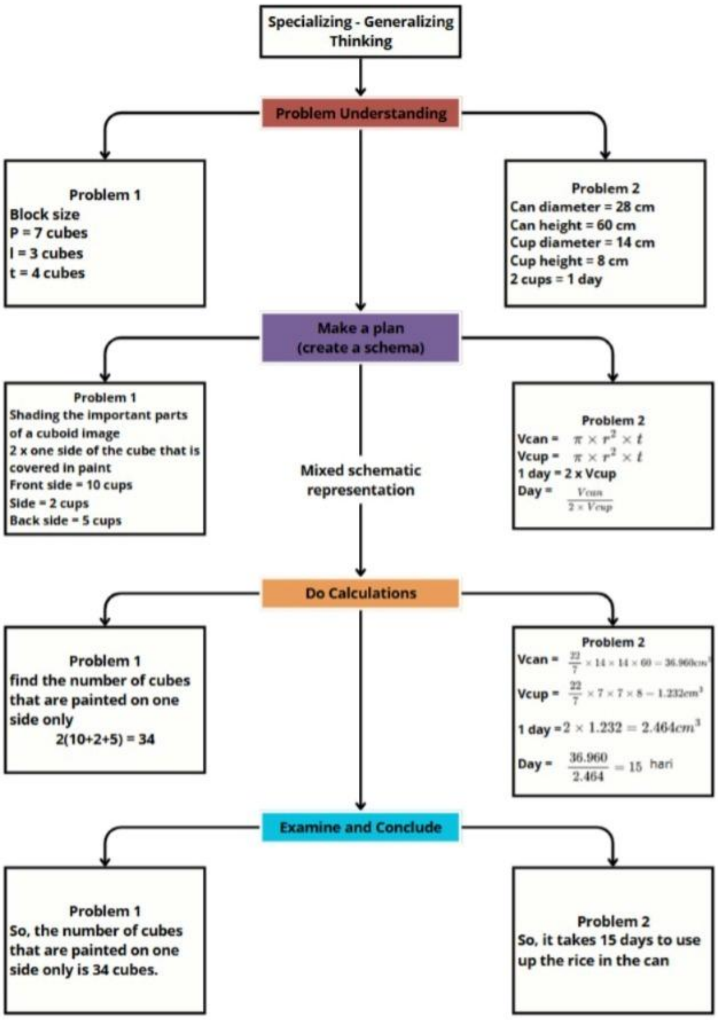

Figure 8. The framework of the thinking process of specializing and generalizing SU2

As a result of the observed thinking process, it can be concluded that schematic representations both in pure and mixed forms play an important role in shaping students' specialization and generalizing thinking flows. SU1 features a more structured and direct approach to presenting the essential elements of the problem, while SU2 incorporates different types of information and visual approaches to support its understanding. These findings are in line with previous studies that have shown that schematic representation helps students summarize important information, organize solution strategies, and connect geometric concepts into new contexts. Such representations serve not only as cognitive aids, but also as bridges between concrete experiences and abstract understandings.

The results of the research conducted on the two subjects show that the subjects are able to solve geometric problems using Pure schematic representation and Mixed schematic representation. Previously, schematic representation was considered quite effective, as it could evoke specializing-generalizing thinking when solving spatial geometry problems. Various research results also indicate that the presence of Pure schematic representation and Mixed schematic representation will have a clear positive effect on overall student performance, and most students successfully reused the representations encountered to solve new problems (Anwar et al., 2021). This statement is in line with the explanation by Estes & Wisniewski, which states that schematic structures are built based on concepts and principles rooted in previous experiences. This creates connections between different concepts (Anwar, 2023). By developing schemas, students can better understand questions and problems (Anwar et al., 2021). Fagnant & Vlassis reinforce this argument by stating that schematic representations play a crucial role in solving complex problems, as they allow students to summarize the essence of the problem in schematic forms (Anwar & Rahmawati, 2022).

These findings have positive implications for mathematics education, particularly in curriculum development and learning. The curriculum should provide space for students to form and use schemas in understanding geometry concepts, as well as encourage teachers to integrate visual and verbal approaches in teaching. However, this research still has limitations, especially in terms of the number of subjects and the scope of topics. Further research can extend the object of study to other mathematical materials, test the effectiveness of representation quantitatively, and explore the use of digital technology to enrich the form of schematic representation in learning.

CONCLUSION AND SUGGESTIONS

Based on the results and discussions presented regarding the resolution of geometric problems by exploring the thinking ability of specializing-generalizing, there are two processes in specializing-generalizing thinking, namely Pure schematic representation for subject 1 (SU1) and Mixed schematic representation for subject 2 (SU2). Pure schematic representation is a representation that presents information in a concise, clear, and structured manner. This representation typically includes diagrams or images that directly depict the key elements of the problem without unnecessary additional information. Mixed schematic representation is a representation that combines various types of information, often including visual elements and text.In this approach, the information may not be well-structured, which can lead to confusion.

From both thinking processes, a specializing-generalizing thinking method was found, so it is hoped that this research can assist educators in designing meaningful learning. This can enhance the specializing-generalizing thinking ability of students. This research is also expected to inspire educators in creating learning tools, learning media, and evaluation tools that are appropriate to the needs and cognitive levels of students.

This study revealed that students face significant difficulties in transitioning from specializing to generalizing in spatial geometry. To address this, educators should implement structured scaffolding techniques and explicit instructional models that support students in developing generalization skills.

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