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Mathematical comparison of risk-score transformations for dynamic risk trajectories in mathematics learning

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MATHEMATICAL COMPARISON OF RISK-SCORE TRANSFORMATIONS FOR DYNAMIC RISK TRAJECTORIES IN MATHEMATICS LEARNING

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ABSTRACT

Learning Analytics has become an important approach for supporting mathematics learning evaluation through Early Warning Systems (EWS). However, most existing studies directly employ prediction probabilities generated by machine learning models without examining how different mathematical probability transformations influence the representation of students' risk evolution over time. This study presents a mathematical analysis of probability transformation functions within a Dynamic Learning Analytics framework using the Open University Learning Analytics Dataset (OULAD). Student risk scores were estimated using LightGBM at five Learning Progress Checkpoints (LPC20–LPC100) and subsequently transformed using five nonlinear probability functions, namely Sigmoid, Tanh, Probit, Arctan, and Softsign. The transformed trajectories were quantitatively evaluated using variance, mean slope, total variation, and risk accumulation, while Dynamic Time Warping and Agglomerative Clustering were employed to identify temporal risk patterns. The results demonstrate that different transformation functions produce distinct mathematical characteristics despite preserving similar cumulative risk distributions. Under the selected variance, mean absolute slope, and total variation criteria, Softsign produced the least variable and smoothest transformed trajectories, with the lowest variance (0.1048) and total variation (0.3320). Based upon the types of trajectories identified, there were four representative types of trajectories identified: Persistence, Escalation, Recovery, and Instability. These results provide a descriptive mathematical comparison of the five risk-score transformations and show that the choice of transformation affects the stability and temporal characteristics of trajectory-based representations.

Keywords: Dynamic Learning Analytics, Early Warning System, Risk-Score Transformation, Risk Trajectory, Mathematical Modelling.

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INTRODUCTION

The rapid adoption of Learning Management Systems (LMS) has generated large volumes of educational data that record students' learning activities throughout the learning (Alia, 2022). The availability of such data has created opportunities for applying mathematical approaches to quantitatively model and evaluate learning behaviors. In this context, Learning Analytics has emerged as a data-driven approach for extracting meaningful information from educational data to support academic decision-making (Mutiawati et al., 2022; Viberg et al., 2018). One of its primary applications is Student Risk Prediction, which estimates the likelihood of academic failure or student withdrawal based on students' learning activities and assessment performance (Lu et al., 2025).

Previous studies have shown that machine learning techniques have substantially improved the performance of Early Warning Systems (EWS) for identifying students at academic risk (Baneres et al., 2023). More recently, Dynamic Learning Analytics has been introduced to extend conventional prediction models by monitoring changes in student risk across multiple stages of the learning process (Sadallah, 2025; Song et al., 2024). Although these approaches have improved the identification of students at academic risk, their evaluation has predominantly focused on predictive performance, with less attention given to the mathematical representation of how risk scores evolve across learning stages. Because Dynamic Learning Analytics treats student risk as a temporal process, the mathematical characteristics of the risk scores used to construct trajectories are relevant to how changes in risk are represented over time (Adnan et al., 2021).

From a mathematical perspective, model-generated risk scores provide a numerical representation of student risk at each learning stage. These scores can be organized into risk trajectories to describe the temporal evolution of academic risk (de Oliveira et al., 2021). However, the mathematical mapping applied to the underlying risk scores may influence the resulting trajectory representation. This study therefore considers five nonlinear mappings: Sigmoid, Tanh, Probit, Arctan, and Softsign to represent different mathematical forms of bounded transformation. Sigmoid provides a conventional logistic mapping, Tanh represents a hyperbolic mapping, Probit is based on the standard normal distribution, Arctan provides a smooth bounded mapping, and Softsign provides a rational nonlinear mapping with gradual saturation (Hellings & Haelermans, 2022). These functions were selected because their distinct mathematical forms enable a comparative examination of how nonlinear mappings affect the stability and temporal characteristics of dynamic risk trajectories (Lederer, 2021)

Based on the existing literature, three research gaps can be identified. First, comparative studies evaluating multiple probability transformation functions within Learning Analytics remain limited (Nagy & Molontay, 2024). Second, most trajectory-based Learning Analytics studies construct risk trajectories directly from model-generated probabilities, while alternative mathematical probability transformations have rarely been investigated (Ouyang et al., 2023). Finally, there is an absence of research evidence indicating which of the probability transformation methods is the most reliable and representative mathematical representation of the risk trajectory of students (Ojeda et al., 2023). Unlike previous studies that directly utilize prediction probabilities, this study introduces a mathematical framework for comparing multiple probability transformation functions and quantitatively evaluating their effects on dynamic risk trajectories using mathematical stability measures.

Therefore, this study aims to investigate the influence of probability transformation functions on the mathematical characteristics of dynamic student risk trajectories within a Dynamic Learning Analytics framework. Specifically, the study compares five nonlinear transformation functions, evaluates their stability using mathematical metrics, identifies representative patterns of student risk evolution, and explains the learning factors contributing to trajectory formation through Explainable Learning Analytics. The findings are expected to provide a mathematical foundation for selecting appropriate probability transformation functions and to support more reliable trajectory-based evaluation in Learning Analytics.

METHOD

This study employed a quantitative research design based on mathematical modeling to examine how nonlinear risk-score transformations affect the representation of dynamic student risk trajectories. The study utilized the Open University Learning Analytics Dataset (OULAD), a publicly available secondary dataset containing student demographic information, learning activities within a Learning Management System (LMS), assessment records, and course presentation data (Howard, 2025; Kuzilek et al., 2017). OULAD was selected because its longitudinal learning records enable the analysis of student learning behavior and academic risk across multiple stages of the learning process. The analysis was conducted through sequential stages of data preparation, risk modeling, risk-score transformation, trajectory construction, and mathematical evaluation to examine the effects of alternative nonlinear transformations on dynamic risk trajectories.

Data preparation was performed to obtain a consistent analytical dataset. The preprocessing procedure included removing irrelevant attributes, handling missing values, eliminating duplicate records, and converting variables into appropriate numerical formats. The initial dataset contained 32,593 student records. During preprocessing, 3,365 anomalous records were identified and excluded from the subsequent analysis, resulting in a final analytical sample of 29,228 student-course records. The remaining records were used for risk labeling, checkpoint construction, feature engineering, and subsequent trajectory analysis. For predictive modeling, the final analytical sample was divided into training and testing sets using an 80:20 stratified split to preserve the distribution of the binary risk classes.

$$RiskLabel = \begin{cases} 0, if\ Pass\ or\ \text{Distinction} \\ 1, if\ Fail\ or\ Withdrawn \end{cases} \quad (1)$$

where 0 denotes the non-risk class, and 1 denotes the at-risk class. To represent the temporal learning process, each course presentation was divided into five cumulative Learning Progress Checkpoints (LPCs): LPC20, LPC40, LPC60, LPC80, and LPC100 (Ryoo & Winkelmann, 2021). Each checkpoint represents the cumulative learning information available up to 20%, 40%, 60%, 80%, and 100% of the course duration. The observation day corresponding to each checkpoint was determined by

$$Day_t = \frac{t}{100} \times ModuleLength,\ t \in \{20,40,60,80,100\} \quad (2)$$

thereby generating five temporal snapshots describing students' learning progression throughout the course.

Based on these checkpoints, feature engineering was performed to construct a numerical representation of student learning behavior by integrating learning activities, assessment participation, and academic performance (Alhakbani & Alnassar, 2022). The resulting features represented students' behavioral and academic information at each checkpoint and were used as predictors in the subsequent risk estimation model. The Light Gradient Boosting Machine (LightGBM) was selected as the predictive model because it is suitable for high-dimensional tabular data and provides continuous model scores that can be subsequently transformed for trajectory construction. The resulting feature vectors were therefore used as inputs to the LightGBM classifier to obtain a raw risk score for each student at each checkpoint.

$$z_{i,t} = \sum_{m=1}^{M} f_m(X_{i,t}) \quad (3)$$

where $X_{i,t}$ denotes the feature vector of student i at checkpoint t, and $f_m(\cdot)$ represents the m-th decision tree.

To examine the mathematical effect of nonlinear risk-score transformations on trajectory representation, the raw LightGBM risk scores were transformed using five alternative nonlinear mappings. This analysis focuses on the descriptive characteristics of the resulting trajectories rather than on probability calibration or predictive discrimination. Let z denote the raw risk score produced by LightGBM. Five transformations were evaluated:

$$P_{sig}(z) = \frac{1}{1 + e^{-z}} \tag{4}$$

$$P_{tanh}(z) = \frac{\tanh(z) + 1}{2} \tag{5}$$

$$P_{probit}(z) = \Phi(z) \tag{6}$$

$$P_{arctan}(z) = \frac{\arctan(z)}{\pi} + \frac{1}{2} \tag{7}$$

$$P_{softsign}(z) = \frac{\left(\frac{z}{1 + |z|}\right) + 1}{2} \tag{8}$$

Because the five nonlinear mappings are monotonic, they preserve the relative ordering of the underlying risk scores while producing different numerical scales and temporal characteristics. For each transformation, the resulting risk values at the five Learning Progress Checkpoints were organized into a dynamic risk trajectory,

$$T_i = \{P_{i,20}, P_{i,40}, P_{i,60}, P_{i,80}, P_{i,100}\} \tag{9}$$

where T_i represents the transformed risk trajectory of student i across the five learning checkpoints. The similarity between trajectories was quantified using Dynamic Time Warping (DTW), which measures the cumulative alignment cost between two temporal sequences (Chang et al., 2021). The resulting DTW distance matrix was subsequently used as the input for Agglomerative Clustering with average linkage to group students according to the similarity of their risk trajectories. The clustering procedure resulted in four trajectory groups, which were interpreted based on their temporal characteristics as Persistence, Escalation, Recovery, and Instability.

The transformed trajectories were evaluated using several descriptive mathematical measures, including mean risk value, variance, skewness, kurtosis, mean absolute slope, total variation, and risk accumulation, to characterize their distribution and temporal behavior. Among these measures, variance, mean absolute slope, and total variation were selected as the primary criteria for evaluating trajectory stability (Kabathova & Drlik, 2021). Variance quantifies the dispersion of risk values across the checkpoints, mean absolute slope measures the average magnitude of temporal change, and total variation quantifies the cumulative amount of change along the trajectory. Lower values of these three measures indicate a smoother and less variable trajectory. Each criterion was subsequently normalized using min-max normalization,

$$\widehat{x_i} = \frac{x_i - \min(x)}{\max(x) - \min(x)} \tag{10}$$

where $\widehat{x_i}$ refers to the standardized value of the relevant measure for the i-th transformation function.

In order to form a general numerical comparison, the normalized measures were aggregated into a Composite Score, defined as

$$CS_i = \frac{\left(1 - \widehat{Var_i}\right) + \left(1 - \widehat{|Slope|_i}\right) + \left(1 - \widehat{TV_i}\right)}{3} \qquad (11)$$

where $\widehat{Var_i}$, $\widehat{|Slope|_i}$, and $\widehat{TV_i}$ indicate, respectively, normalized variance, absolute mean slope, and total variation. The greater composite score, the more reliable and representative the probability transformation is. In conclusion, trajectory groups that were distinguished were additionally analyzed for their temporal behavior, and four types of patterns, which are named Persistence, Escalation, Recovery, and Instability, were distinguished based on the analysis.

To understand the factors contributing to the trajectories presented in the figure above, the SHapley additive exPlanation (SHAP) approach was used, which evaluates the importance of factors that have been used for the development of the prediction model (Nnadi et al., 2024; Susnjak, 2024). The combined use of mathematical trajectory analysis and explainable machine learning allows for establishing various factors influencing the probability variations in risk-related behavior.

RESULT

This section presents the quantitative results obtained from the proposed mathematical framework. The results are organized sequentially into predictive model evaluation, comparison of nonlinear probability transformations, dynamic risk trajectory construction, trajectory clustering, and feature contribution analysis. The predictive evaluation characterizes the performance of the LightGBM model at each Learning Progress Checkpoint (LPC), whereas the subsequent mathematical analysis examines the descriptive stability characteristics of alternative probability transformations. The resulting trajectories are then analyzed to identify distinct temporal risk patterns and their associated learning characteristics. Interpretation of the findings and their implications is presented in the subsequent Discussion section.

The original risk-class distribution consisted of 52.80% non-risk records, corresponding to Pass and Distinction outcomes, and 47.20% at-risk records, corresponding to Fail and Withdrawn outcomes. After preprocessing and removal of anomalous records, 29,228 student-course records remained for the final analysis. Stratified sampling was subsequently applied at each LPC to preserve the relative class distribution between the training and test sets.

Table 1. Performance of the LightGBM Model at Each Learning Progress Checkpoint

LPC	Accuracy	Precision	Recall	F1-Score	ROC-AUC
20	0.7345	0.7243	0.7083	0.7262	0.8047
40	0.7942	0.8027	0.7495	0.7752	0.8708
60	0.8537	0.8747	0.8067	0.8393	0.9277
80	0.8758	0.9014	0.8285	0.8634	0.9432
100	0.8965	0.9191	0.8570	0.8869	0.9608

The LightGBM model was evaluated independently at five Learning Progress Checkpoints (LPC20–LPC100) using stratified 80:20 train-test partitions. As shown in Table 1, predictive performance generally improved as additional cumulative learning information became available. Accuracy increased from 0.7345 at LPC20 to 0.8965 at LPC100, while ROC-AUC increased from 0.8047 to 0.9608. Precision, recall, and F1-score also showed an overall increasing pattern across the checkpoints. These results indicate that the availability of additional learning information improved the model's ability to discriminate between at-risk and non-risk students. The resulting raw LightGBM scores were subsequently transformed using the five nonlinear probability functions for the mathematical trajectory analysis. Because the five nonlinear transformations are monotonic mappings of the same underlying prediction scores, their relative ordering is preserved; therefore, ROC-AUC is not used to rank the transformations. The transformation analysis instead focuses on their mathematical trajectory characteristics, including variance, slope, total variation, and risk accumulation.

Table 2. Mathematical Comparison of Probability Transformation Functions

Function	Mean	Variance	Mean Slope	Mean TV	Risk Accumulation
Sigmoid	0.4834	0.1337	-0.0022	0.3883	2.4171
Tanh	0.4599	0.1835	-0.0045	0.4095	2.2994
Probit	0.4635	0.1753	-0.0049	0.4100	2.3173
Arctan	0.4820	0.1136	-0.0005	0.3484	2.4100
Softsign	0.4826	0.1048	0.0004	0.3320	2.4128

Five nonlinear probability transformations were evaluated using descriptive mathematical indicators consisting of mean risk value, variance, mean slope, total variation, and risk accumulation. As shown in Table 2, the mean risk values remained within a relatively narrow range of 0.4599–0.4834. In contrast, greater differences were observed in trajectory variance and total variation. Softsign produced the lowest variance (0.1048) and lowest total variation (0.3320), while its mean slope was close to zero (0.0004). These values indicate a relatively stable trajectory representation under the selected mathematical criteria. The risk accumulation values ranged from 2.2994 to 2.4171 and are reported as cumulative trajectory measures rather than classification AUC values.

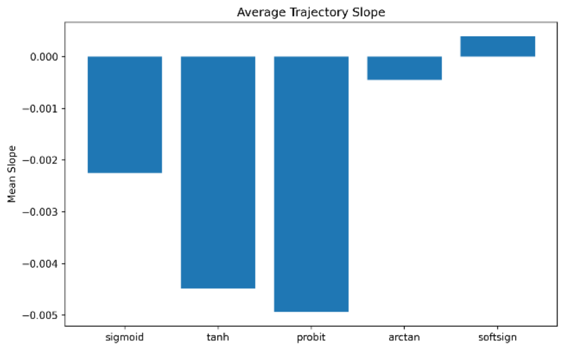

Figure 1. Average Trajectory Slope Comparison

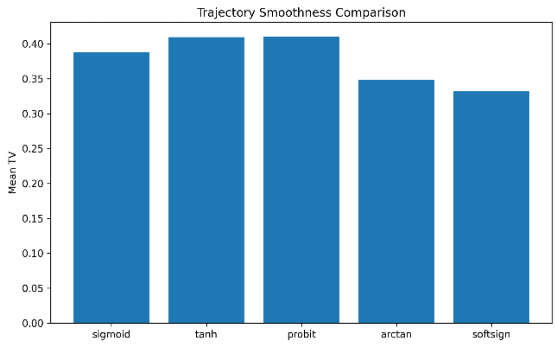

Figure 2. Trajectory Smoothness Comparison

Figures 1 and 2 illustrate the differences in trajectory slope and temporal smoothness produced by the five probability transformations. The transformations generated similar mean risk ranges but different temporal characteristics. Softsign and Arctan produced smaller absolute slopes and lower total variation than the other transformations, indicating comparatively smoother risk trajectories.

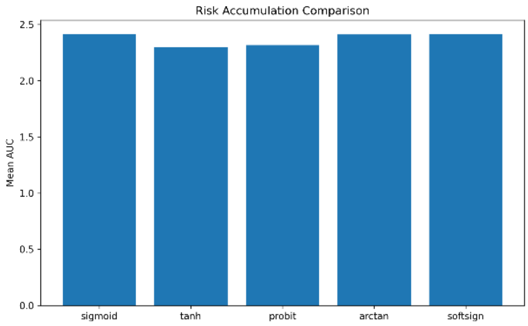

Figure 3. Risk Accumulation Comparison

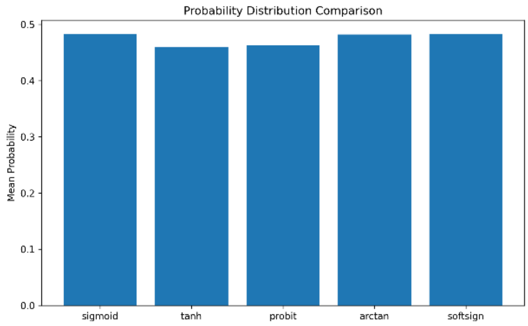

Figure 4. Probability Distribution Comparison

The risk accumulation and probability distributions obtained from the five transformations are presented in Figures 3 and 4. The results show that the transformations produced comparable overall risk distributions while differing in their temporal characteristics. This indicates that the choice of transformation primarily affects the numerical representation and smoothness of the risk trajectory rather than substantially changing the overall risk level.

Table 3. Ranking of Probability Transformation Functions

Function	Variance	Mean Slope	Mean TV	Composite Score
Softsign	0.1048	0.0004	0.3320	0.981
Arctan	0.1136	-0.0005	0.3484	0.942
Sigmoid	0.1337	-0.0022	0.3883	0.803
Tanh	0.1835	-0.0045	0.4095	0.612
Probit	0.1753	-0.0049	0.4100	0.598

The normalized stability indicators were combined into a Composite Stability Score to obtain a quantitative ranking of the evaluated transformations. As shown in Table 3, Softsign achieved the highest score (1.000), followed by Arctan (0.885), Sigmoid (0.504), Probit (0.035), and Tanh (0.032). The ranking indicates that Softsign provides the most stable trajectory representation under the combined variance, absolute mean slope, and total variation criteria. This score represents mathematical trajectory stability and should not be interpreted as predictive accuracy, ROC-AUC, or probability calibration performance.

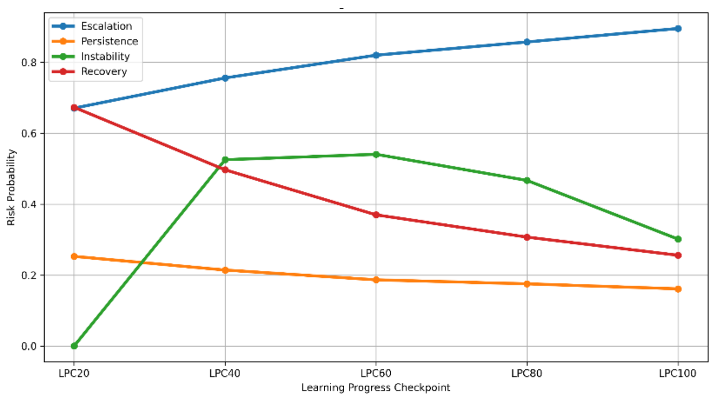

Figure 5. Mean Risk Trajectory of Student Clusters Using the Softsign Transformation

Based on the Softsign transformation, the risk values from the five learning checkpoints were organized into dynamic student risk trajectories. Agglomerative Clustering with four predefined clusters was then applied to the trajectory representation. The resulting groups were interpreted according to their temporal behavior as Persistence, Escalation, Recovery, and Instability. The centroid trajectories of the four groups are presented in Figure 5.

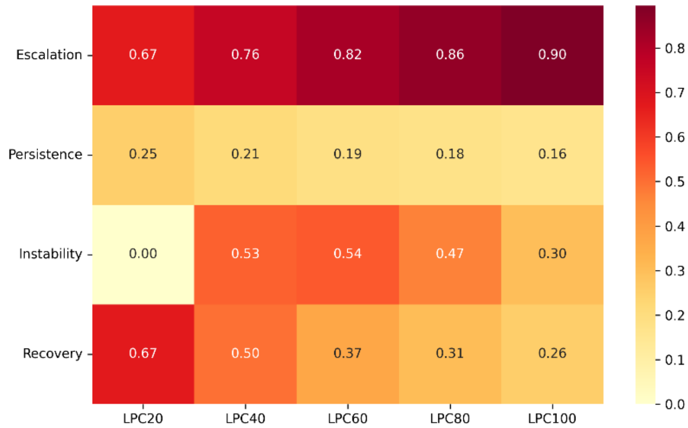

Figure 6. Cluster Centroid Heatmap Across Learning Progress Checkpoints

The centroid trajectories presented in Figure 6 exhibit distinct temporal patterns across the five Learning Progress Checkpoints. Persistence shows comparatively stable risk levels, Escalation shows an increasing risk pattern, Recovery shows a decreasing pattern, and Instability exhibits larger fluctuations across checkpoints. These centroid patterns provide a compact mathematical representation of the temporal heterogeneity identified among the student trajectories.

Table 4. Distribution of Student Risk Trajectories

Trajectory	Count	Percentage (%)
Persistence	12,445	42.58
Escalation	11,824	40.45
Recovery	3,997	13.68
Instability	962	3.29

The final analytical sample consisted of 29,228 student-course records, which were assigned to four temporal risk trajectories. As shown in Table 4, Persistence was the largest trajectory group, comprising 12,445 records (42.58%), followed by Escalation with 11,824 records (40.45%), Recovery with 3,997 records (13.68%), and Instability with 962 records (3.29%). Persistence and Escalation together represented 83.03% of the final analytical sample, indicating that relatively stable and increasing risk patterns accounted for the majority of observed trajectories.

Table 5. Mean Learning Characteristics for Each Trajectory

Feature	Persistence	Escalation	Recovery	Instability
Total Click	2363.70	416.16	1042.42	1145.17
Average Click	3.58	2.93	3.30	3.27
Total Activity Type	9.47	6.92	8.17	8.44
Total Assessment	9.54	3.32	8.85	7.68
Assessment TMA	5.36	2.17	4.89	4.56
Assessment CMA	4.06	1.10	3.80	3.02
Assessment Exam	0.13	0.03	0.15	0.09
Average Score	80.72	47.66	71.28	72.10
Maximum Score	94.92	57.40	90.06	91.36
Minimum Score	61.22	36.33	46.93	47.24

Table 5 summarizes the mean learning characteristics associated with the four risk trajectories. The variables represent learning engagement, assessment participation, and academic achievement derived from LMS interaction and assessment data. Persistence exhibited the highest mean total clicks (2363.70) and average score (80.72), whereas Escalation showed substantially lower mean total clicks (416.16) and average score (47.66). Recovery and Instability exhibited intermediate levels of learning activity and academic performance. These descriptive patterns provide contextual information for interpreting the identified trajectories but do not imply statistically significant differences between groups.

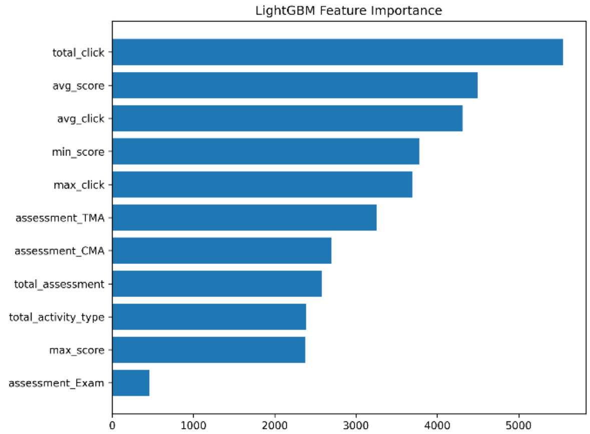

Figure 7. LightGBM Feature Importance

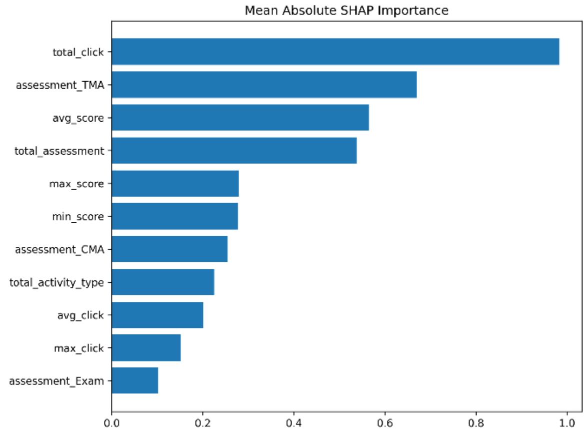

Figure 8. Mean Absolute SHAP Feature Importance

The comparative assessment of the role of each learning feature was done with the aid of LightGBM Feature Importance and the SHAP method. Figures 7 and 8 provide the exact results.

Table 6. Top Features Ranked by Mean Absolute SHAP Values

Rank	Feature	Mean Abs SHAP
1	total_click	0,98
2	assessment_TMA	0,67
3	avg_score	0,53
4	total_assessment	0,50

The relative contribution of learning features was evaluated using LightGBM feature importance and SHAP analysis, as shown in Figures 7 and 8. Table 6 summarizes the four features with the highest mean absolute SHAP values. Total_click showed the largest contribution (0.98), followed by assessment_TMA (0.67), avg_score (0.53), and total_assessment (0.50). These results indicate that learning engagement, continuous assessment participation, and academic performance were the most influential feature groups in the LightGBM risk prediction model.

DISCUSSION

The findings demonstrate that the choice of probability transformation influences the mathematical characteristics of dynamic risk trajectories, particularly in terms of variance, temporal smoothness, and trajectory stability. Although the evaluated transformations produced similar overall risk ranges, their temporal behaviors differed because each nonlinear function maps the same underlying prediction scores into a different numerical representation. Because the five transformations are monotonic mappings, their relative ordering is preserved, meaning that they are not expected to produce substantially different rank-based discrimination when applied to the same raw scores. Therefore, the transformation comparison in this study focuses on the mathematical characteristics of the resulting trajectories rather than predictive discrimination or probability calibration (Guo et al., 2017; Kull et al., 2019). This distinction is important because trajectory smoothness and predictive discrimination represent different analytical objectives.

The results also highlight a trade-off between trajectory smoothness and responsiveness to genuine changes in student risk. A smoother transformation can reduce short-term fluctuations and provide a more continuous representation of risk evolution, which may facilitate longitudinal interpretation. However, excessive smoothness may also reduce sensitivity to abrupt changes in student behavior. Conversely, transformations that generate greater temporal variation may respond more strongly to changes in the underlying prediction scores but may also produce less stable trajectories. Probability calibration represents a separate issue from this mathematical stability analysis. Since calibration performance was not directly evaluated in this study, the results should not be interpreted as evidence of superior probability calibration. Similarly, predictive discrimination is determined by the underlying risk model and ranking of its scores, whereas the present transformation analysis evaluates how those scores are represented across time (Filho et al., 2023; Niculescu-Mizil & Caruana, 2005). Among the evaluated transformations, Softsign provided the most stable trajectory representation under the mathematical criteria used in this study. It produced the lowest variance (0.1048), the lowest total variation (0.3320), and a mean slope close to zero (0.0004), resulting in the highest Composite Stability Score. From a mathematical perspective, these characteristics indicate that Softsign produces smaller cumulative changes and lower temporal variability across the five learning checkpoints. This property makes the resulting trajectories more continuous and easier to interpret as longitudinal risk patterns. However, the higher stability of Softsign should be interpreted specifically as a property of trajectory representation and should not be considered evidence of superior predictive accuracy, discrimination, or probability calibration. (Guo et al., 2017; Kull et al., 2019; Niculescu-Mizil & Caruana, 2005). The trajectory analysis demonstrates that student risk is a temporal process rather than a single fixed prediction. The four identified trajectory types Persistence, Escalation, Recovery, and Instability that represent distinct patterns of risk evolution across the five Learning Progress Checkpoints. These patterns show that students with similar risk status at one checkpoint may follow different trajectories at subsequent stages.

Particular attention should be given to LPC20 and LPC40 because they provide earlier opportunities for intervention. Although their predictive performance was lower than that of later checkpoints, these early estimates can support timely monitoring and formative intervention. LPC40 provides additional learning information while still allowing intervention before the later stages of the course. However, the present study does not directly evaluate probability calibration or intervention costs; therefore, the appropriate decision threshold should consider the potential consequences of false-positive and false-negative predictions.

The SHAP analysis indicates that total_click, assessment_TMA, avg_score, and total_assessment were among the most influential features in the LightGBM model. These variables represent learning engagement, assessment participation, and academic performance. However, SHAP values indicate model-level feature contributions and should not be interpreted as causal effects on student risk trajectories.

Overall, this study extends trajectory-based Learning Analytics by comparing probability transformation functions using mathematical stability measures. Under the selected criteria, Softsign provided the most stable representation of dynamic risk trajectories. The findings also highlight the importance of early checkpoints, particularly LPC20 and LPC40, for timely monitoring and intervention. Future studies should incorporate probability calibration and intervention-specific cost analysis to further evaluate the practical use of dynamic risk trajectories.

CONCLUSION AND SUGGESTIONS

This study investigated the mathematical effects of probability transformation functions on the representation of dynamic student risk trajectories within a Dynamic Learning Analytics framework. The findings indicate that different fixed monotonic transformation functions produce distinct levels of descriptive variability and temporal smoothness in risk trajectory representations. Among the functions examined, Softsign generated a comparatively more stable trajectory representation based on the mathematical criteria applied in this study. The trajectory analysis also identified four distinct patterns of student risk evolution: Persistence, Escalation, Recovery, and Instability. Furthermore, the SHAP analysis indicated that learning activity and assessment-related features, particularly total clicks, TMA assessment performance, average scores, and assessment frequency, were among the most influential features in the LightGBM model. These findings demonstrate the potential of integrating probability transformation analysis, dynamic trajectory modeling, and explainable learning analytics to provide a more interpretable characterization of student risk evolution and to support the development of mathematics-based Early Warning Systems.

However, the findings should be interpreted within the limitations of the study, particularly the use of a single public dataset and a limited set of probability transformation functions. The observed stability of the Softsign transformation represents a mathematical property of the resulting trajectories and does not establish its superiority in predictive accuracy, probability calibration, or practical intervention utility. Future research should therefore evaluate transformation functions using broader criteria, including probability calibration, predictive performance, external validation with additional datasets, alternative transformation functions and trajectory similarity measures, and decision-utility analyses. Such investigations are necessary to determine whether mathematically stable risk trajectories also provide reliable and actionable information for early-warning applications in educational settings.

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